Comparison of different analysis models to measure plastic strains on sheet metal forming parts by digital image processing

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Abstract

The principal strains of sheet metals and their limitations while forming can be obtained by using a strain measurement system. A strain measurement may employ one of two different approaches: namely the total least square optimization method or the multiple regression analysis (MRA) method. With both methods plastic strains of deformed parts are calculated based on the non-deformed reference configuration designated by a circle and the deformed configuration, which is a curve-fitting ellipse. In the MRA method, the mathematical formula is simpler reducing the required computations than that of the total least square optimization method. While the formula has a greater margin of error, this margin proves less than significant in the practical application of the method’s results.

Information from the results of a strain measurement system can be used to determine the sheet metal’s formability and strain path allowing engineers to determine at which point, the sheet metal will crack. They can then change the thickness and the material of the sheet metal, or modify its shape accordingly to maximize the metal’s efficiency. Strain measurement systems possess practical advantages in their actual application; they improve the quality of sheet metal being produced by minimizing defects in sheet metal during production. One industrial case study of fine stamping electronic part is discussed to demonstrate the proposed methodology. © 2003 Elsevier Science Ltd. All rights reserved.

Keywords: Plastic strain measurement; Circular grid analysis; Sheet metal forming; Image processing

1. Introduction

For the past three decades, a technique known as circular grid analysis stood out above all others as the best method to help engineers in the sheet metal industry. With this technique, circular grids are imprinted on the surface of a metal strip, which is then drawn until cracks occur or obvious necking is observed. These circles, under plane stress condition, generally become elliptical as well as measurable. By measuring the deformed grids, the major and minor principal strains can be calculated. Then the data can be plotted in a diagram, known as Forming Limit Diagram (FLD) [1,2]. The FLD consists of three zones as shown in Fig. 1: the safety zone, the failure zone and the marginal zone, which are used for the evaluation of sheet metal formability. FLD has been constructed by means of the stretching test with strips and a hemispherical punch [3]. The width of the strips is changed to generate different strain paths. Two refinements on this test method have been proposed. The first one changes the geometry of the punch into ellipti-
The circular grid system is widely used to evaluate sheet metal formability. It permits immediate and direct measurement of the maximum elongation of the sheet at any location. Grid strain analysis involves etching a pattern of fine circles onto the sheet before pressing as shown in Fig. 2. During pressing, the circles are deformed into ellipses that can be measured to determine major and minor strains produced in the test sample. Then, a reference is made to the FLD, which plots the major and minor strains at fracture enabling an estimate to be made of the criticality of the press forming operation. The strain values and the ratio of major to minor strain give information on the type of deformation in various areas of the press-formed parts; for example, whether the metal has been drawn or stretched. This information provides insight into the press forming operation that can be used to solve problems in the die development work and part design.

Conventional method manually measures the deformed grids with the aid of a transparent tape and magnifier. However, the grids are typically too small to have the principal direction accurately recognized. Consequently, the conventional method is not only tedious but also inaccurate. Harvey [6] has developed an optical strain measurement system to acquire the principal strains of deformed grids automatically with an image processing technology. By using the operation of binary-image thresholding, the dedicated open circle has been transferred into inner and outer edges of the deformed grid through convolution and recognition, and both are determined by least square elliptical regression. Then, the dimension of the deformed grid is obtained by computing the mean dimension of inner and outer edges. Another similar system described in Ref. [7] is limited to solid-circle analysis, however, the processing results are influenced significantly by light level. The author [8,9] has presented an image processing system, in which the strains of local necking grids or crack grids can be measured directly. Schedin and Melander [10] used an image analyzer to evaluate the large strains from industrial sheet metal stamping with a square grid; however, the corners of the deformed square of this system must be identified manually for preventing inaccuracy of processing. Vacher et al., [11] used correlation method in image analysis to study FLD and found that the strain localization may occur early during the forming process revealing the onset of localized necking is not a sudden phenomenon.

Among these papers that are discussed previously, the analysis model used to fit these elliptical grids is multiple regression method. Yet there are some drawbacks in the mathematical model of the multiple regression method. For example, it treats the same variable both as dependent variable and independent variable in its mathematical formulation. Therefore, the principal lengths of the ellipse calculated using the multiple regression analysis (MRA) method are dependent on translations and rotating angles. In the proposed paper, the total least square (TLS) optimization method is studied to overcome these problems. Both methods will be compared and evaluated based on the theoretical data points and the experimental data points acquired from sheet metal forming parts.

2. Theoretical methods

2.1. Image processing

The purpose of image processing is to obtain useful data through the analysis and manipulation of images. This is achieved through the development and implementation of operations upon the images. In digital image processing, the image is represented by numerous discrete points of defined brightness. Each point, called a pixel, has a numeric location and a numeric brightness, i.e. gray level. By manipulating these values of brightness within the image through algorithms, the programming is relatively easy. Image analysis is concerned with the extraction of useful measurement, which is distinguished from other types of image processing, such as coding, restoration and enhancement, in that the ultimate product is usually a numerical output rather than a picture. A mathematical relationship of detected points on an object boundary provides an analytical description of the image [12]. Several methods can be used to establish the mathematical relationship, such as curve optimization (regression analysis) and line-to-point transformation (Hough transformation).

2.2. Multiple regression analysis

The general form of an ellipse can be represented as

$$Ax^2 + Bxy + Cy^2 + 2Dx + 2Ey + F = 0 \quad (1)$$

Without loss of generality, assume that $E$ is not equal to zero, then Eq. (1) can be transformed into
The square of the errors in the N sampling points into Eq. (2). The summation of the square of the errors in the y coordinate can be represented as

\[ SS_T = \sum_{j=1}^{N} (y_j - \alpha - \beta_1 x_j - \beta_2 x_j^2 - \beta_3 x_j y_j - \beta_4 x_j^2 y_j)^2. \]  

Therefore, the regression procedure determines the coefficients of Eq. (4) when SS_T is extreme. This means that the partial differentials of SS_T with respect to all its coefficients are zero, i.e.

\[ \frac{\partial SS_T}{\partial \alpha} = 0 \quad \text{and} \quad \frac{\partial SS_T}{\partial \beta_i} = 0 \quad i = 1,4. \]  

Then the coefficients and the principal axes of the ellipse can be determined [8].

### 2.3. Total least square analysis

Assume that \((u_i, v_i)\) is the coordinate of the data point \(P_i\) acquired in frame \(A\) whose axes are coincided with principal axes of the ellipse, and \((x_i, y_i)\) is one of the points in the ellipse \(E\) of world frame \(B\) as shown in Fig. 3. The TLS formulation then takes the following forms [14]

\[ y = \alpha + \beta_1 x + \beta_2 x^2 + \beta_3 xy + \beta_4 y^2. \]  

Minimize:

\[ SS_T = \sum_{i=1}^{n} (d(p_i, E))^2 \]  

subject to

\[ \frac{\partial h(x,y,\lambda)}{\partial x} = 0, \quad \frac{\partial h(x,y,\lambda)}{\partial y} = 0, \]  

In general cases where \(u \neq 0\) and \(v \neq 0\), Eq. (8) yields

\[ \lambda = \frac{(a^2 - b^2)x + vb^2x - ua^2y}{a^2x^2 + b^2y^2}. \]  

The solutions of Ferraris and Cardan are adapted to solve for \((x, y)\) from Eq. (9) in combination with standard equation of ellipse \(g(x, y) = 0\). In the second step, the Hooke and Jeeves search method [16] and Fibonacci search method [17] are adopted to search for the optimal values of \(a, b, c_x, c_y, \text{and} \ \theta\).

### 3. System implementation and operation

The equipment of the strain measurement system is shown in Fig. 4. The main processor is a PC 586 computer. A Matrox frame grabber was used for the interface of vision system and PC. It has 640 × 480 resolution in one frame. The image-input device, which is mounted on a copy stand, is a black and white Sony TV camera with a zoom lens of 10 times magnification ratio. The ring-type light source is adopted to obtain homogeneous illumination. The relationship between the world coordinate system and the image coordinate system is obtained by camera calibration, which is assumed to obtain the linear transformation. Wang [18] proposed a camera calibration technique to adopt the perpendicular alignment error of optical axes to specimen surface. The proposed image measurement system is implemented in Visual Basic programming language and Matrox image

![Fig. 3. The geometric relation of dedicated ellipse E between frame A, axes coincided with the principal direction of ellipse and frame B, world coordinate system.](image-url)
4. Results and discussion

4.1. Simulation test of data points in a theoretical ellipse

The total sixty data points, located in the contour of a theoretical ellipse whose major and minor lengths and coordinates of center \((a, b, c_x, c_y) = (3, 5, 10, 20)\), are generated by a computer program designed to change the angle increment every \(6^\circ\). In Table 1, five theoretical ellipses with different principal axes’ directions, i.e. \(\theta = 30, 45, 60, 80, 89\) are tested and the simulation results by TLS method and MRA are also listed. The analysis accuracy of TLS method is better than that of MRA method. There are two advantages with the TLS method: (1) the analysis results are close to exact solutions, and (2) its orientation is invariant.

4.2. Simulation test of data points in an acquired ellipse image from sheet metal specimen

The repeatability test is also conducted to verify the variation of measurement results for an elliptic grid indicated in the center of the monitor of Fig. 4. This test is in the deformed sheet metal specimen placed at 10 different orientations for image measurement. The results of the TLS and MRA analysis are shown in Table 2. The resolution of the image system under this magnification ratio is close to 0.015 mm, which is calculated according to the length of a source part at 1.5 mm corresponding to 100 pixels in the image frame. It is concluded that the deviation of data acquired in this system by both methods is small and acceptable, which is close to hardware limitation of the image system. The standard deviation of TLS analysis is less than the MRA, but the difference proves insignificant. The computation time needed in the TLS analysis is much longer than that of the MRA. Therefore, the MRA method is recommended when performing strain measurement of sheet metal forming parts.

4.3. Strain measurement for fine stamping electronic part

A fine stamping electronic part is used to demonstrate the proposed strain system by MRA method as shown in Fig. 6. In order to obtain the strain path in critical or interesting areas of the stamping part, an incremental test is conducted. The procedures are as follows: (1) prepare several processes of source materials; (2) mark fine circular patterns on the surface of the source materials by electro-chemical etching; (3) perform the stamping process by changing the strokes of the mechanical press; and (4) measure the dimensions and plastic strains. The scene to measure the strain of the fine stamping part is depicted in Fig. 7 and these three deformed grids at...
Table 1
Results of simulation test by TLS and MRA methods in a theoretical ellipse

<table>
<thead>
<tr>
<th>Parameters</th>
<th>a (mm)</th>
<th>b (mm)</th>
<th>c_x (mm)</th>
<th>c_y (mm)</th>
<th>q (degree)</th>
<th>SS T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact value</td>
<td>3</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>2.460675 × 10^{-12}</td>
</tr>
<tr>
<td>TLS method</td>
<td>3.000001</td>
<td>4.999999</td>
<td>9.999999</td>
<td>20.000001</td>
<td>30.000004</td>
<td>2.765542 × 10^{-11}</td>
</tr>
<tr>
<td>MRA method</td>
<td>2.999996</td>
<td>4.999994</td>
<td>9.999998</td>
<td>20.000000</td>
<td>30.000003</td>
<td>7.201242 × 10^{-10}</td>
</tr>
<tr>
<td>Exact value</td>
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<td>5</td>
<td>10</td>
<td>20</td>
<td>45</td>
<td>2.439085 × 10^{-12}</td>
</tr>
<tr>
<td>TLS method</td>
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<td>5.000000</td>
<td>10.000000</td>
<td>20.000000</td>
<td>45.000001</td>
<td>3.082317 × 10^{-11}</td>
</tr>
<tr>
<td>MRA method</td>
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<td>10.000000</td>
<td>20.000000</td>
<td>44.999996</td>
<td>2.278954 × 10^{-10}</td>
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</table>

Table 2
Results of simulation test by TLS and MRA methods in an acquired ellipse image (in mm)

<table>
<thead>
<tr>
<th>Sample no.</th>
<th>MRA analysis</th>
<th>TLS analysis</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Length of</td>
<td>Length of</td>
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<tr>
<td></td>
<td>major axis</td>
<td>minor axis</td>
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<tr>
<td>1</td>
<td>1.696</td>
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<tr>
<td>5</td>
<td>1.652</td>
<td>1.623</td>
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<td>6</td>
<td>1.684</td>
<td>1.585</td>
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<td>10</td>
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<tr>
<td>Mean</td>
<td>1.679</td>
<td>1.585</td>
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<tr>
<td>Standard deviation</td>
<td>0.015</td>
<td>0.026</td>
</tr>
</tbody>
</table>

Fig. 6. A fine stamping electronic part (at three steps) is used to demonstrate the proposed strain measurement system.

Fig. 7. Screen for measuring strain of the fine stamping part.

seven deformation steps are shown in Fig. 8. The grid size in diameter and the distance between the two neighboring grids are 0.4 mm. The specimen was drawn until punch displacement reached the preset value. The strain paths of three different grids at seven steps are depicted in Fig. 9 by using the proposed strain measurement image processing system. The measured results are close to the strain paths of deep drawing parts, i.e. the absolute values of compressive strains are equal to tensile strains. The experimental results are close to theoretical ones, which shows the acceptance of the proposed measurement system.

5. Conclusions

A strain measurement system for sheet metal forming by digital image processing technology and its fabrication process are presented. There are two different approaches that are studied, namely the TLS optimization method and the MRA method. The simulation test
of data points in a theoretical ellipse reveals two advantages of TLS method: (1) the analysis results are close to exact solution, and (2) its orientation is invariant. After simulation, test of data points in an acquired ellipse image from specimen shows that the deviation of data acquired in this system is small and acceptable, and the difference between TLS and MRA methods is insignificant. Comparing to the TLS optimization method, the MRA method is simpler and faster within reasonable accuracy.

**Fig. 8.** Three deformed grids at seven steps in an incremental test.

**Fig. 9.** The measured strain paths of three different grids depicted in Fig. 7.

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**References**