Composite Approach to High Strain-Rate Stress-Strain Curve of Reactive Powder Concrete

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ABSTRACT: In this study, we investigate the stress-strain curves of reactive powder concrete (RPC) at high strain rates by using SHPB test, the Burgers’ mechanics model and secant-moduli method. The RPC composites are examined at the age of 10 days containing RPC mortar as the matrix, and three volume concentrations, 1\%, 2\% and 3\%, of steel fiber as the inclusion respectively. Strain rates including 1x10\textsuperscript{2} - 1x10\textsuperscript{3}/sec by means of SHPB are applied to RPC. From the reproduced stress-strain curves of RPC matrix at a given strain rate by Burgers’ model, a secant-moduli method is adopted to predict stress-strain curves of RPC composites. Results indicate that dynamic four-parameter model with parameters, \(k_1(\dot{\varepsilon}), k_2(\dot{\varepsilon}), \eta_1(\dot{\varepsilon}),\) and \(\eta_2(\dot{\varepsilon}),\) can well simulate the stress-strain curves of RPC without steel fibers. The proposed composite method is capable of predicting the stress-strain relations of RPC composites in high strain-rate regimes, especial for strain rates approaching to 1000/sec.

1. INTRODUCTION

Systemic research for dynamic concrete mechanics has been more than 20 years. Three diameters of impact bar for SHPB (split-Hopkinson pressure bar) with 51mm, 76mm and 100mm have been adopted, respectively. Tedesco et al \cite{1} chose 51mm in diameter SHPB to investigate dynamic behavior of concrete, and found that concrete materials are sensitive to strain rate. As the strain rate applied to concrete is greater than a threshold of strain rates, the strength increases linearly with increasing strain rates. Rose et al \cite{2} examined 51\,mm\times51\,mm cylindrical specimens of concrete under different material ages and curing time by applying 100 - 10\textsuperscript{3}/sec. Some projectile impact tests have been carried out to determine the impact resistance of high-strength concrete by Zhang et al. \cite{3}, and they disclosed that, in order to increase the compressive strength, a reduction in the water-to-cement ratio and the reduction of the coarse aggregates were essential. Many researches for dynamic behaviour of RPC have been also reported \cite{4}.

From the theoretical side, most theoretical studies \cite{5-6} conducted thus far, however, did not take the advantage that concrete is essentially a composite material consisting of the cementitious binder and the aggregates. As such, the properties of the concrete, such as the initial Young’s modulus, the peak strength and peak strain, and the stress-strain relations under a constant strain-rate loading, are closely related to the properties of cementitious binder, water-to-binder ratio, and volume fraction of the aggregate, among others. This observation has prompted us to study dynamic strain-rate sensitive concrete using the concept of composite materials. Here, we use SHPB to measure stress-strain relations of reactive powder concrete (RPC) at high strain rates, and propose a theoretical approach to determine stress-strain curves in terms of strain rates, especial for strain rates within 10\textsuperscript{2} - 10\textsuperscript{3}/sec.

2. EXPERIMENTS

2.1 Mixture proportion

RPC consists of RPC mortar as the matrix and steel fiber as the inclusions. The constituents of RPC mortar are Type II Portland cement (PC) with 15.8\,\mu m particle size, silica fume (SF) with 0.1-0.2\,\mu m, quartz powder (QP) with 5-20\,\mu m, quartz sand (QS) with 200-600\,\mu m, and ASTM Type-G superplasticizer (SP). Three volume concentrations \(c_1\) of the steel fiber are used. The length and the diameter of steel fiber are 30mm and 0.5mm and a shape similar to a spheroid with an aspect ratio (length-to-diameter ratio) of \(\alpha = 60.\)

Mixture proportions of RPC with water-to-binder ratio (w/b) of 0.19 are shown in Table 1, where SCF0, SCF1, SCF2 and SCF3 are referred to the RPC containing \(c_1=0\%\), 1\%, 2\% and 3\% steel fibers in volume, respectively.

<table>
<thead>
<tr>
<th>Material</th>
<th>w/b</th>
<th>PC</th>
<th>SF</th>
<th>QP</th>
<th>QS</th>
<th>SP</th>
<th>Steel fiber (c_1) (vol %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCF0</td>
<td>0.19</td>
<td>714</td>
<td>216</td>
<td>252</td>
<td>944</td>
<td>36</td>
<td>0</td>
</tr>
<tr>
<td>SCF1</td>
<td>0.19</td>
<td>714</td>
<td>216</td>
<td>252</td>
<td>918</td>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>SCF2</td>
<td>0.19</td>
<td>714</td>
<td>216</td>
<td>252</td>
<td>891</td>
<td>36</td>
<td>2</td>
</tr>
<tr>
<td>SCF3</td>
<td>0.19</td>
<td>714</td>
<td>216</td>
<td>252</td>
<td>865</td>
<td>36</td>
<td>3</td>
</tr>
</tbody>
</table>

2.2 Material preparation

RPC was cast into steel moulds and compacted using an iron surcharge to consolidate the RPC specimen. The specimens were stored in room temperature for 2days, then removed from the moulds, and cured in 90\,\degree C water for 7 days. After that, the RPC specimen was placed in the air to dry off for 24 hours before the tests. Five specimens for each RPC material are prepared to be tested.

2.3 Loading and fracture pattern

Dynamic strain rates were performed by using SHPB setup on cylinder specimen with 50mm in diameter and 25mm in length. SHPB setup to study here is depicted in Fig. 1, including gas launcher, projectile, Hopkinson bar and data analyzing apparatus. The projectile impact on the Hopkinson bar develops a compressive longitudinal incident wave transmitted into the RPC specimen, and a small part is reflected back to the Hopkinson bar due to the difference of impedances. The whole process of wave propagation was recorded via strain gauges and data analyzing apparatus. RPC specimen with strain gauges lies in between the Hopkinson bar shown in Fig. 2.

Figure 1  Split-Hopkinson pressure bar (SHPB) apparatus
From the incident wave of impact bar, we can calculate strain rate applied to the specimen. Several designed strain rates $\dot{\varepsilon}$ including $1 \times 10^2 - 1 \times 10^3$/sec were chosen to generate the stress-strain curves of the RPC mortar (SCF0 material) and RPC composites (SCF1, SCF2 and SCF3 material). Fig. 3 is SCF2 specimen applied 338/sec strain rate during SHPB test photoed by high speed camera. Figs. 4-7 are RPC fragments after high strain-rate loading.

3. MODIFIED BURGERS MODEL FOR RPC

In order to develop the composite approach, we shall first use the stress-strain relations of RPC mortar at a given strain rate. Here, the rate-dependent, nonlinear viscoelastic behavior of RPC mortar will be reconstructed by a modified Burgers model as depicted in Fig. 8. The four-parameter Burgers model has been found sufficiently to model the cement property at six orders of magnitude of the strain rate from $10^{-6}$ to $10^{-1}$/sec [7]. But when the tests span over high strain–rate such as $10^2-10^3$/sec in this study, rate-dependent Burgers model still needs to be verified with the experiment.

The modified Burgers’ model with such rate-dependent properties are shown in Fig. 8, where the rate-dependent spring and dashpot constants are denoted as $k_1(\dot{\varepsilon}), k_2(\dot{\varepsilon}), \eta_1(\dot{\varepsilon})$, and $\eta_2(\dot{\varepsilon})$, respectively. Under a constant strain-rate loading, the governing differential equation for such a modified model can be established as

$$\ddot{\delta}_1(t) + \frac{k_1}{\eta_1 + \frac{k_2}{\eta_2}} \dot{\delta}_1(t) + \frac{k_2}{\eta_1 \eta_2} f(t) = \left(\frac{k_1 k_2}{\eta_1 \eta_2}\right) w$$

(1)

where $f(t) = \text{external load}$, $t = \text{loading time}$, $w = \text{loading velocity}$ given by the sum $\Delta t$, $\Delta t$. Following the previous study [7], the rate-dependent stress-strain relation can be generalized to
\[
\frac{\sigma}{f_p} = 3.71 \left[ e^{m(\epsilon - 0.03)} - \left( 1 + 0.27 \eta \left( \frac{L}{A} \right) e^{m(\epsilon - 0.03)} \right) \right] + \eta \frac{L}{A} f_p
\]

where \( \sigma \) and \( f_p \) = stress and peak stress of RPC, respectively, and \( A \) and \( L \) = area and length of the specimen, in turn. \( m_1 \) and \( m_2 \) are the characteristic roots in terms of four spring and dashpot constants. The stress-strain relation of RPC given in Eq. (2) can account for the effects of strain rate, peak stress, and the size of the specimen.

We have used this constitutive equation in Eq. (2) to simulate the tested stress-strain data of RPC mortar (RPC without steel fibers, SCF0) at various strain rates shown in Fig. 9, where the dotted lines are simulated by the modified Burgers model.

\[
\kappa = \frac{1}{1 + c_1(p_1/p_2)} \quad \mu = \frac{1}{1 + c_1(q_1/q_2)}
\]

In Eq. (3), the units of spring and dashpot constants for \( k_1, k_2, \eta_1, \) and \( \eta_2 \) are \( 10^3 \text{N/m}, 10^7 \text{N/m}, 10^4 \text{N*s/m} \) and \( 10^9 \text{N*s/m} \) respectively. With these constants and Eq. (2), the simulated and test results of RPC mortar shown in Fig. 9 have a close agreement related at a given strain rate. They can provide sufficiently accurate description for the constitutive behavior of RPC mortar in the composite model, that is, allow us to reproduce the stress-strain relations and calculate secant moduli simultaneously.

### 4. COMPOSITE APPROACH FOR RPC COMPOSITES

While the cementitious materials are under monotonically increasing load, the secant-moduli approach is particularly suitable for the calculation of the nonlinear stress-strain curves of the cement mortar [7]. To address this composite approach, we first recall the overall effective bulk and shear moduli of the two-phase composite with spheroidal inclusions and expressed by [8]

\[
\kappa = \frac{1}{1 + c_1(p_1/p_2)} \quad \mu = \frac{1}{1 + c_1(q_1/q_2)}
\]

where parameters \( p_1, p_2, q_1, \) and \( q_2 \) depend on the moduli of the constituent phases, and volume concentration and shape of the inclusions through Estelby’s S tensor.

For the calculation of nonlinear stress-strain relations of the composite, the secant moduli will replace the elastic modulus for the matrix and the composite. In other words, we replace \( \kappa_0 \) and \( \mu_0 \) by \( \kappa^s \) and \( \mu^s \), respectively, in the parameters \( p_1, p_2, q_1, \) and \( q_2 \) in Eq. (4), at a given strain and strain rate. This procedure allows us to calculate the secant moduli, \( \kappa^s \) and \( \mu^s \), of RPC composites. After \( \kappa^s \) and \( \mu^s \) have been determined, the secant Young’s modulus of the composite can be solved from the isotropic connection

\[
E^s(\varepsilon, \dot{\varepsilon}) = \frac{9k^s(\varepsilon, \dot{\varepsilon})\mu^s(\varepsilon, \dot{\varepsilon})}{3k^s(\varepsilon, \dot{\varepsilon}) + \mu^s(\varepsilon, \dot{\varepsilon})}
\]

At a given strain rate, the axial stress-strain curve of the composite then follows from

\[
\sigma(\varepsilon, \dot{\varepsilon}) = E^s(\varepsilon, \dot{\varepsilon}) \varepsilon(\varepsilon, \dot{\varepsilon})
\]

where \( \sigma \) and \( \varepsilon \) are the stress and strain of the composite, respectively.

For RPC containing steel fibers (RPC composites), the RPC mortar (matrix) intrinsically possess a nonlinear viscoelastic matrix dependent on strain rate, but mechanical properties of the steel fiber (inclusion) are considered to be linear and independent of the strain rate according to the previous report in [9].

To calculate the secant elastic of RPC composites, we need to use the secant moduli instead of elastic moduli of the matrix in Eq. (4). The secant moduli of RPC mortar (SCF0) can be generated from modified Burgers model and Eq. (3) by applying the same as strain rates of RPC composites. Besides, the Young modulus and the Poisson ratio of steel fiber we use here are 200 GPa and 0.3 respectively.

### 5. RESULTS

We compare the stress-strain curves of RPC determined from the composite approach with the experiments. The results for SCF1, SCF2 and SCF3 under dynamic strain rates are shown in Figs. 10-12, Figs. 13-15, and Figs. 16-17 in turn. These figures in dotted lines are strict predictions-not simulations-by the secant moduli approach.

Results indicate that the composite approach can capture the dynamic strain-rate sensitivity of RPC sufficiently well, especial for strain rates near 1000/sec depicted in Figs. 12, 15 and 17.
Figure 12  Comparisons for SCF1 under 1056/sec strain rate

Figure 13  Comparisons for SCF2 under 554/sec strain rate

Figure 14  Comparisons for SCF2 under 804/sec strain rate

Figure 15  Comparisons for SCF2 under 1039/sec strain rate

Figure 16  Comparisons for SCF3 under 663/sec strain rate

Figure 17  Comparisons for SCF3 under 978/sec strain rate

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REFERENCES


