An Investigation of the Construction for Parametric Spur Gears Using Computer Solid Modeling Techniques

Wern-Kueir Jehng*  Hwi-Long Chang**  Ping-Chang Lee***

Abstract

The development of solid modeling has created numerous techniques for unambiguous representations of complex geometric objects. In this paper, a solid modeling package, PRO/Engineering developed by Parametric Technology Corporation, is used to construct and generate a variety of tooth profiles for spur gears. Their geometries, and gearing primitives are created. A general mathematical parametric model proposed by Chang and Tsai is used. It includes three kinematic parameters: rotating angle, pressure angle, and polar distance which are relative to a first order constraint differential equation. If the pressure angle is specified as the function of rotating angle, then the relative kinematic properties can be studied. Using polynomial functions and their kinematic parametric relations, a series of spur gears is developed. The present study spots how the tooth geometries can be synthesized and constructed using computer solid modeling techniques, which are very important for CNC machining.

Keyword : Spur Gears, Computer Solid Modeling, Kinematic Parameters Method, CNC Machining.

1. Introduction

In the science graphic technologies, computer-based systems for modeling the geometric solid objects that are used to depict spatial phenomena, object graphic contour, computer vision manufacturing product design, and virtual reality simulation are becoming more and more important. The main structure of the systems are symbol representations designating abstract solids in the subsets of Euclidean space that model physical solids [1]. Representations are the sources of data for procedures which compute useful properties of objects. These three dimensional solid geometry is vital to the industries to produce discrete products. A useful and reliable computer geometry system must have these following components [2]: (l) symbol structure that represents solid objects; (2) processes that use such representations for answering geometric questions about the objects (for example “what is the distance between two points?, what is the volume?”); (3) input facilities
that mean for creating and editing object representations and for working processes; and (4) output facilities and representations of results. The subsystem which provides facilities for entering, storing, and modifying object representations is called a geometric modeling system (GMS) that inherits a branch of mathematics to deal with shape and spatial relations, calculates automatically the mass properties of defined object, or produces perspective view with hidden lines suppressed or dashed. Most of the GMS currently used in computer-aided design and manufacturing (CAD/CAM) possess relatively sophisticated human-engineered input facilities to modify and add the missing information and to resolve inconsistencies. Unambiguous representations play a central role in the applications of computer science that ensure a GMS having sufficient information to support a unique geometric contour representation.

Today's parametric solid modeling systems inherit feature-based design characteristics and provide attractive environment to designers [3]. Such a user-defined feature (UDF) enables designers to build their own feature library. In the feature-based modeling systems, feature definitions must specify how the feature instance is constructed on the basis of values of attributes. Therefore, a new design for the geometric model of the part is easily created by combining the feature instance geometries, then merged to a complete solid object. In general, there are three dominant solid representations in use [4] such as constructive solid geometry (CSG), boundary representation, and spatial subdivision. There are other representation schemata as well which are less widely used. Any solid representation should admit the unambiguous and algorithmic determination of point membership. Any point in space, it must be able to determine algorithmically whether the point is inside, outside, or on the surface of a solid. A solid usually are represented as unions of surfaces, with each surface represented in terms of its boundary, plus data defining the surface in which the face space lies. They construct complete, valid and unique representations that automatically guarantee the unambiguity of solid objects. In this paper, we use PRO/Engineering 2002 version to aid design parametric spur models. The module PRO/surface [5] gives the ability to create a variety of surface features and entities that generate versatile gear faces and their space location information. This information is created with PRO/E surface tools versus that of typical solid commands. In addition, the gear surfaces are turned into a solid object then the computer solid gear models are developed. The detailed techniques are depicted in Section 3.

Concerning gear theories and kinematics [6], spur gears and helical gears are rotated by two parallel rotating axes which are classified as the plane mechanism, bevel gears are applied to two intersecting shafts that are defined as the spherical mechanism, while the hypoid gears and worm gears which transfer two non-intersecting and non-parallel axes are deemed as the spatial mechanism. By the meshing theory, Leonard Euler [7] worked out the law of conjugate action, in which the actions of two mesh tooth profiles can be studied from the driving of a cam and the follower. The common normal of the meshing
tooth profiles should pass through a fixed point, for example, the spur gearing mesh must pass through the pitch point. Gears designed according to this law have a steady speed ratio. On gearing conjugate theories, the conjugate tooth profile is an envelope of a gear profile or a rack profile under a configurated motion. Litvin [8] had proposed the meshing equation indicating that the common normal of two conjugate gear profiles at a contact point should be perpendicular to the relative velocity of the couple gears. The popular gearing involute and cycloidal curves are the substantial conjugate profiles that have widely been used in spur gear. They can be constructed by solid modeling and described by ordinary graphic method. An involute curve is generated by a tracing point carried by a line rolling on a circle, while a cycloidal curve is generated by a tracing point attached to a circle rolling on the pitch circle of the gears. To analyze and derive spur gear mainly depends on contour tooth geometries, while its conjugate tooth profiles are based on the rack cutters, or imaginary racks with the law of gearing or the meshing equations. In spur gear, the involute profiles have distinguished advantages in having a flexible geometry that allows small center distance error, and a great amount of interchangeability between individual gears. The relative theories are depicted in section 2. Three examples are illustrated and demonstrated in section 3. Section 4 concludes this article.

2. Mathematical models of spur gear tooth profiles

In general, spur gears are divided into two families, the involute and noninvolute. The involute gears are popularly used today with distinguished advantages. However, the involute profiles belong to the meshing couple of convex / convex surface contact that induces a great contact stress, and a small number of gear tooth is confronted with the undercutting that decreases the bending strength at the root of tooth. Therefore, noninvolute gear has been proposed, and a number of research articles have been published. Lebeck and Redzimousky, Kasuya, Eggeman, and Hlebanja [9, 10, 11, 12] mainly focus on increasing the load capacity. The designed noninvolute gear synthesized in the form of convex / concave profiles reduce the contact stress, increase the thickness near the root of tooth and improve the meshing condition and efficiency of lubrication. In this section, we have focused on introducing the mathematical functions and the construction procedures of the solid gear models.

2-1 Introduction involute functions of spur gear

Spur gear has an almost infinite number of forms that can be constructed as gear tooth profiles. However, the involute profile is the most common gear-tooth forms that are used to transmit power. In order to appreciate the great simplicity of the involute form both in its theory and in its manufacture, it is necessary to learn a clear understanding of the principles of conjugate gear-tooth action. In all situations when two curved surfaces act against each other, the line of action between them will be along the common normal to the two curves at the point of tangency between them. The two rotating axes transmit uniform rotary motion, and the values of the
momentary pitch radius remain constant for all operating positions of the contacting profiles.

It is convinced that uniform rotary transmission from one axis to another by means of gear teeth, it is necessary that the normal to the profiles to these teeth at all points of contact must pass through a fixed point in the common center line of the two axes. This fixed point in the common center line is called the pitch point. Figure 1 shows the base circle, and an involute curve starting at point \( a \), with a typical point \( A \) at radius \( r \). The normal to the involute at \( A \) touches the base circle at \( b \).

![Involute curve and its profile angle and roll angle](image)

Angle \( \alpha' = \beta - \theta \) is defined as profile angle. The radius \( Ob \) is perpendicular to \( bA \), since \( bA \) touches the base circle, and \( Ob \) is therefore parallel to the involute tangent vector \( V \). Hence, the angle \( bOA \) is equal to the profile angle and from the properties of involute curve, the following relationships can be derived.

\[
\text{angle } bOA = \alpha' \tag{1}
\]
\[
\text{arc } ab = bA \tag{2}
\]
\[
r \cos \alpha' = R_s \tag{3}
\]
\[
bA = R_s \tan \alpha' \tag{4}
\]

The roll angle at the radius is defined as \( \beta \), which is the angle between the radius through \( a \) and the involute tangent at \( A \). Since \( Ob \) is parallel to the involute tangent at \( A \), the angle \( boa \) is equal to the roll angle, then the length of arc \( ab \) can be represented as Eq.(6).

\[
\text{angle } boa = \beta \tag{5}
\]
\[
\text{arc } ab = R_s \beta \tag{6}
\]

Combining Eqs.(2), (4) and (6), we have,

\[
\beta = \tan \alpha' \tag{7}
\]

The angle between the radii \( OA \) and \( Oa \) is clearly a function of \( \alpha' \), and the involution function is defined as \( \text{inv} \alpha' \), then the relationship between angles \( \alpha' \) and \( \beta \) can be derived as:

\[
\theta = \text{inv} \alpha' = \beta - \alpha' = \tan \alpha' - \alpha' \tag{8}
\]

Equation 8 is used to develop the geometry of an involute gear. When \( \alpha' \) is given, the value of \( \text{inv} \alpha' \) can be calculated by Eq.(8). Contrarily, if \( \alpha' \) is unknown, it is necessary to find \( \alpha' \) which Polden [13] proposed the following equations to solve the problem.

\[
q = \left(\text{inv} \alpha'\right)^{1/3} \tag{9}
\]
\[
\frac{1}{\cos \alpha'} = 1.0 + 1.04004q + 0.32451q^2
\]
\[
-0.00321q^3 - 0.00894q^4
\]
\[
+ 0.00319q^5 - 0.00048q^6 \tag{10}
\]

Colbourne [14] verified the above two equations solved by two steps, the maximum error is \( 0.0001 \), for the value of \( \alpha' \) between \( 0^\circ \) and \( 65^\circ \), and this range of \( \alpha' \) values is sufficient for most practical proposes. Next step, for the purpose of evaluating the geometry of tooth profiles, it is necessary to use the radius \( r \) to
specify any point A unwound from point a on the tooth profile. The profile angle at A is then derived from equation 3.

\[ r = \frac{R_s}{\cos \phi_a} \] .................................(11)

The angle between line OA and the fixed line Oa is expressed by the involute function

\[ \text{angle } A\text{Oa} = \text{inv} \phi' = \tan \alpha' - \alpha' \] ..........(12)

Equation (12) is the famous polar equation of the involute curve that had been further studied by Buckingham [15]. The curve is described by the end of a line that is unwound from the circumference of a circle, as shown Figure 2.

![Figure 2. Involute curve of a circle](image)

The circle from which the string is generated is called the base circle. The equations of the involute can be deduced as:

\[ \theta = \beta - \tan \frac{1}{2} \sqrt{r^2 - R_s^2} \] ..................(13)

\[ R_s \beta = \text{arc } ab = \text{length of } bA = \sqrt{r^2 - R_s^2} \]

then

\[ \beta = \frac{\sqrt{r^2 - r_s^2}}{R_s} \] .................................(14)

Therefore,

\[ \theta = \frac{\sqrt{r^2 - R_s^2}}{R_s} - \tan \frac{1}{2} \sqrt{r^2 - R_s^2} \] ..............(15)

Equation (15) is most commonly employed to design the involute curve function.

2-2 Introduction of the noninvolute function of spur gears

A general mathematical model of parametric tooth profiles for spur gears had been proposed by Chang and Tsai [16]. It is derived by using plane kinematics and differential geometry with three kinematic parameters: rotation angle, polar angle, and polar distance. By the gearing kinematics, a first order constraint differential equation is deduced, which strictly correlates the three kinematic parameters. Therefore, the tooth geometries and geometric properties can be described by these parametric functions. The polar angle is conveniently specified as the function of rotation angle. Figure 3 shows that a tooth profile \( \Sigma_{a} \) meshes with the tooth profile \( \Sigma_{a'} \) and the initial contact position is located at the pitch point P.

When gear 1 rotates counterclockwise with an angular displacement \( \phi_1 \) about the center \( O_1 \), gear 2 will rotate clockwise with an angular displacement \( \phi_2 \) about the center \( O_2 \). The contact point, in the instance, will move to \( P_1 \) and \( P_2 \) that are the coinciding points respectively, located at the tooth profiles of gear 1 and gear 2. The unit vector of \( \overrightarrow{PP} \) is normal to the meshing profiles at the contact point, and the measured length of \( \overrightarrow{PP} \)
where $P = R_i i_j$, $n = \cos \alpha i_j + \sin \alpha j_i$, $i_j$, and $j_i$, denote the unit vectors of $X_i$ and $Y_i$ axes, respectively.

When both the meshing tooth profiles of $\Sigma_{id}$ and $\Sigma_{ia}$ are considered, the contact point should be inside the pitch circle of gear 1. Then, the polar angle $\alpha$ is equal to $\frac{\pi}{2} + \alpha'$ ($\alpha'$ is pressure angle) and should be limited in the range:

$$\frac{\pi}{2} \leq \alpha \leq \pi$$ .............................(17)

The left side of Equation (16) can be represented by the following equation.

$$P + \lambda n = (R_j + \lambda \cos \alpha) i_j + \lambda \sin \alpha j_i$$ .............................(18)

In differential geometry and kinematics, differentiating Eq.(16), the total differential equation is given as

$$k_j \times P_j \, d \phi_i + tds_1 = n \, d \lambda + \lambda \, (k_j \times n) \, d \phi_i - \lambda \, \kappa_i \, tds_1$$ .............................(19)

In equation (19), the unit vector $k_j = i_j \times j_i$, $\kappa_i$ and $s_i$ are the curvature and curvilinear length of the tooth profile $\Sigma_{id}$ respectively, and the unit vector $t$ is the common tangent of the meshing tooth profiles at the contact point $P_i$ or $P_2$.

The right side of Eq.(16), where $P = R_i i_j$, $R_i$ is constant, therefore the differential of $R_i$ is zero. Then the right side of Eq.(16) becomes:

$$(d\lambda) \, n + \lambda \,(dn) = n \, d\lambda + \lambda \, (k_j \times n) \, d \phi_i - \lambda \, \kappa_i \, tds_1$$

By the differential geometry [17], $dn = -\kappa_i \, tds_1 + k_j \times n \, d \phi_i$, the result of differentiating Eq.(16) is yielded to Eq.(19). For simplifying Eq.(19), it can be multiplied by a normal unit vector at both sides. By the theory of triple unit vector product, $n \cdot n = 1$, $n \cdot k = 0$, $n \cdot t = 0$ and the vector operation theories, the following relationships can be derived.

$$k_j \times P_j = \begin{vmatrix} i_j & j_i & k_i \\ 0 & 0 & 1 \\ (R_j + \lambda \cos \alpha) & \lambda \sin \alpha & 0 \end{vmatrix}$$

$$k_j \times n = \begin{vmatrix} i_j & j_i & k_i \\ 0 & 0 & 1 \\ \cos \alpha & \sin \alpha & 0 \end{vmatrix}$$

Then $n \cdot (k_j \times n) = (\cos \alpha i_j + \sin \alpha j_i)$

$$= -\sin \alpha i_j + \cos \alpha j_i$$

Therefore, $n \cdot P_j \, d \phi_i$ can be yielded as

$$d\lambda = n \cdot (k_j \times P_j) \, d \phi_i$$

$$= (\cos \alpha i_j + \sin \alpha j_i) \cdot (\cos \alpha i_j + \sin \alpha j_i) \cdot d \phi_i$$

$$= \cos \alpha \, i_j \cdot d \phi_i$$

$$+ (R_j + \lambda \cos \alpha) \, j_i \cdot d \phi_i$$
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\[ \begin{align*}
\alpha + \lambda \sin \alpha \cos \alpha + R_i \sin \alpha \phi \, d \phi_i \\
\lambda = R_i \sin \alpha \, d \phi_i \\
\frac{d \lambda}{d \phi_i} = R_i \sin \alpha \quad \text{........................................ (20)}
\end{align*} \]

Equation (20) is the first order differential equation that strictly defines the relation among the kinematic parameters \( \alpha \), \( \lambda \) and \( \phi_i \) proposed by Chang and Tsai [16]. For clear explanation, it is derived by the above logically sequential steps. If two of the three kinematic parameters are related by a function, then the third kinematic parameter can be determined by the first order differential equation. If \( \mathbf{P}_{id} \) is represented as the initial position vector of the dedendum part tooth profile of gear 1 and is set as the pitch position \( \mathbf{P} \), then the position vector \( \mathbf{P}_j \) of the contact point tooth profile \( \Sigma_{id} \) is gotten by the position vector \( \mathbf{P}_{id} \) of the tooth profile counterclockwise with an angular displacement \( \phi_i \). Therefore \( \mathbf{P}_{id} \) can be represented as

\[ \mathbf{P}_{id} = \mathbf{R}(k_1, -\phi_i) \mathbf{P}_i \quad \text{................. (21)} \]

where \( \mathbf{R}(k_1, -\phi_i) \) is a rotation matrix, \( \mathbf{P}_i \) vector rotates counterclockwise about \( k_1 \) axis. Substituting Eq.(16) into Eq.(21), the tooth profile of \( \Sigma_{id} \) can be derived as

\[ \mathbf{P}_{id} = \begin{bmatrix} R_i \cos \phi_i + \lambda \cos(\alpha - \phi_i) \\
-R_i \sin \phi_i + \lambda \sin(\alpha - \phi_i) \\
0 \end{bmatrix} \quad \text{.............. (22)} \]

In the meshing processes, the tooth profile \( \Sigma_{id} \) of gear 2 is always in conjunction with the tooth profile \( \Sigma_{id} \) of gear 1. Since \( \mathbf{P}_i \) and \( \mathbf{P}_2 \) denote the same vector in the \( S_2(\mathbf{X}, \mathbf{Y}, \mathbf{Z}) \) coordinate system, then the position vector \( \mathbf{P}_2 \) can be represented by the \( S_2(\mathbf{X}, \mathbf{Y}, \mathbf{Z}) \) coordinate system, and the \( \mathbf{P}_2 \) position vector can be represented as

\[ \mathbf{P}_2 = -R_2 \mathbf{i}_2 + \lambda \mathbf{n} \quad \text{........................................ (23)} \]

Differentiating Eq. (23), the total differential equation is yielded as:

\[ -\mathbf{k}_2 \times \mathbf{P}_2 \, d\phi_2 + \mathbf{t} \, ds_2 = \mathbf{n} \left( d\lambda \right) - \lambda \left( \mathbf{k}_2 \times \mathbf{n} \right) \, d\phi_2 - \lambda \kappa_2 \mathbf{t} \, ds_2 \quad \text{........................................ (24)} \]

Where \( \mathbf{k}_2 \) is the unit vector of \( \mathbf{Z}_2 \), \( \kappa_2 \) and \( s_2 \) are the curvature and curvilinear length of tooth profile \( \Sigma_{2s} \), respectively, \( \phi_2 \) is the rotation displacement of gear 2 which is related to \( \phi_i \) as

\[ m_{2i} = \frac{\phi_2}{\phi_i} \quad \text{........................................... (25)} \]

where \( m_{2i} \) is the gear speed ratio.

The tooth \( \Sigma_{2s} \) is represented by the position vector \( \mathbf{P}_{2a} \) that is obtained by rotating the position vector \( \mathbf{P}_2 \) with an angular displacement \( \pi - \phi_2 \). Then in the \( S_2(\mathbf{X}, \mathbf{Y}, \mathbf{Z}) \) coordinate system, \( \mathbf{P}_{2a} \) can be defined as

\[ \mathbf{P}_{2a} = \mathbf{R}(k_2, \pi - \phi_2) \mathbf{P}_2 \quad \text{........................................ (26)} \]

Substituting Eq. (22) into Eq. (25), the tooth profile \( \Sigma_{2s} \) can be deduced as:

\[ \mathbf{P}_{2a} = \begin{bmatrix} R_2 \cos \phi_2 - \lambda \cos(\alpha + \phi_2) \\
R_2 \sin \phi_2 - \lambda \sin(\alpha + \phi_2) \\
0 \end{bmatrix} \quad \text{........ (22)} \]

and the kinematic parameter is related and constrained by the following first order differential equation

\[ \frac{d \lambda}{d \phi_2} = R_2 \sin \alpha \quad \text{........................................ (27)} \]

3. Illustrations of computer solid spur gear sets
The geometric design of a solid spur gear is used as an example for illustration. Before the gear profiles can be determined, it is necessary to specify a few parameters by which the axes will rotate in accordance with the requirements of the system. In Figure 4, the coordinate systems $S(X,Y,Z)$ and $S(X'Y'Z')$ are the two Cartesian systems.

The coordinate systems $S_1(X_1,Y_1,Z_1)$ and $S_2(X_2,Y_2,Z_2)$ are rigidly connected to gear 1 and gear 2, respectively. Let point $P$ be the pitch contact point when there are two meshing gears. Their angular velocities are $\omega_1$ and $\omega_2$ and the speed ratio is $m_{12} = -\frac{\omega_2}{\omega_1}$. The coupled gears rotate at different directions.

Figure 7 shows an involute gear. In this section, how the involute gear is created will be depicted in detail. The involute polar Eq.(15) is applied to create the involute curve and sweep a beautiful involute surface to construct the gear boundary. By the procedure of PRO/E actual operating system, it can be completed by the following steps:

1. Setting the initial dimension of the gear blank is shown in Figure 5. It is a quarter circle block that will be cut involute gear profiles. A center hole is provided to pin a shift in the gear center. The detail dimensions are shown in Fig. 5.

2. Create a reference datum named DTM4, which is a plane through axis A1 counterclockwise above DTM2 by 20°. Then use the system command to create an advanced surface by the variant section sweep method. Sketch an unwound range carve on both sides of the gear blank. The sketched concentric arc is 14° and starts 3° off the DTM4. Use a centerline to present the 3° angle, and avoid the 3° angle snap to 0°. Zoom in or change...
the sketching accuracy to 0.5 if necessary. Make sure that the starting points on both sides are also on the lower end of the arc. If it is not the lower end of the arc, the sketch made has a selection. Feature Tools can change the position of starting point. Complete the concentric arcs on both surfaces, then finish the spine by choosing done. The second trajectory curve is drawn by the next step.

3. Return to the default view and select the front face as the sketching plane for the second trajectory. The second trajectory will have the same dimensions as the first. Don’t start the second sketch exactly on top of the first; let the final modification of the dimension cause them to coincide in the sketching view. Referring to the default view may help you complete the sketch. Choose done and finish the second trajectory arc. After doing the above procedures, the system turns the gear blank to its side view to define the line of cross-section. Sketch a horizontal line above the sketched point 68.00 mm from axes A1 and align the ends to the faces of the blank. This line is the involute curve sweeping line to the other side. Then, choose clone and complete the step.

4. Choose the sketch column command and pick the edit relation icon. The system shows a note book pad to key in the program. From section 2, Eq.(15) shows the famous polar involute curve function as:

\[
\theta = \frac{\sqrt{r^2 - R_s^2}}{R_s} - \tan^{-1}\left(\frac{\sqrt{r^2 - R_s^2}}{R_s}\right) \quad \text{...................(28)}
\]

where \(r\) is the variable unwind involute curve length from the original point shown in Fig. 2. From Eqs.(28) and (29), key in the following program into the editor pad:

\[
r = R_s \left[1 + \left(1 + \left(\frac{\sqrt{r^2 - R_s^2}}{R_s}\right)^2\right)^{-\frac{1}{2}}\right] \quad \text{...................(29)}
\]

Here, \(r\) base is \(R_s\) in Fig 2, ‘unwind’ is the term of \(\sqrt{r^2 - R_s^2}/R_s\), which is the radian value and must be changed to degree, therefore product ‘todeg’, and the line: unwind * todeg – atan(unwind) = trajpar * 14.0 has the same meaning as in Eq.(28), where the trajpar arc length is defined 14\(^o\) in the above step, and for ‘unwind’ is a loop calculation for the next statement. After completing the programming, the left side name of last line of the program must match the system default name in the Pro/E procedures. then save the file and complete the step, the sd7 will snap to the base circle with a radius 64.0 mm and expand a beautiful involute surface shown as in Figure 6.
Figure 6. Two involve curves cut off and build the gear blank profile

Now, return to the default view, create a second surface by the copy mirror command. Create an extrude cut using the front face of the part as the sketching plane and pick the DTM4 as the top reference plane. Sketch a concentric arc with radius 67.0 mm length of radius between the two involute surface and use edge on the boundary of both involute surface to change edge to sketching line. Then, trim the entities if necessary.

5. Finish the feature and look at the default view. Create a single round feature that includes four edges; the two edges at the tips of the tooth and the two edges at the root of the tooth. Use an edge of 1.0 to round the four edges.

6. Create a local group including all of the features, it must be added to the initial parts, including DTM4 through the involute contour and the four rounds. Modify the angle of DTM4 to 5° off the DTM2. Then, pattern the group using the Pattern command from the group menu by incrementing the angle of DTM4. Use 20° between groups and create the total of five groups. Round all other edges of the model using the value of 1.0. Then shade the model a final time to view the result as in Fig. 7. The computer solid gear is thus completed.

Figure 7. The completed computer solid gear

3.2 Example 2: an involute spur gear developed by approach method

A gear has a module of 8 mm and a pressure angle of 20° and the number of teeth is 24. The diameter of the tip circle is 212 mm and the tooth thickness is 14.022 mm. The gear mate a pinion with speed ratio of 0.5. To derive the computer solid model for the couple of gears, the following step are developed.

1. Calculate the following basic dimension of the gear by the equations derived in section 2.

   The standard pitch circle radius
   \[ R = \frac{1}{2} N m = \frac{1}{2} \times 24 \times 8 = 96 \text{mm} \]

   The base circle radius,
   \[ R_b = R \cos \alpha = 96 \cos 20 = 90.210 \text{mm} \]

   The standard circular pitch,
   \[ P_r = \pi m = 25.133 \text{mm} \]

   The base circular pitch,
   \[ P_{b} = \frac{2\pi R_b}{N} = 23.617 \]
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\[ \text{inv} \alpha' = \tan \alpha' - \alpha' = \tan 20 - 20 \times \frac{\pi}{180} \\
= 0.014904 \]

\[ \alpha' = \cos^{-1}\left(\frac{R_s}{R_t}\right) = 31.675' \]

\[ \text{inv} \alpha' = \tan \alpha' - \alpha' = 0.064175 \]

The thickness of top gear face is

\[ t_r = R_s \left[ \frac{t}{R_t} + 2 \left( \text{inv} \alpha' - \text{inv} \alpha' \right) \right] \]

\[ = 106 \left[ \frac{14.022}{96} + 2 \left( 0.014904 - 0.064175 \right) \right] \]

\[ = 5.036855\text{mm} \]

2. Calculate the radius of \( r \) when the unwind angle is expanded from \( 1' \) to \( 20' \).

The values in table 1 are calculated by the following steps:

Table 1. The relative data used to construct Fig. 9 and Fig. 10

<table>
<thead>
<tr>
<th>Unwind angle ( \theta' )</th>
<th>Pressure angle</th>
<th>Unwind radius</th>
<th>Position value ((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1'</td>
<td>32.54</td>
<td>107.072</td>
<td>1.869</td>
</tr>
<tr>
<td>2'</td>
<td>29.89</td>
<td>104.038</td>
<td>3.631</td>
</tr>
<tr>
<td>3'</td>
<td>26.37</td>
<td>100.703</td>
<td>5.270</td>
</tr>
<tr>
<td>4'</td>
<td>21.25</td>
<td>96.824</td>
<td>6.754</td>
</tr>
<tr>
<td>5'</td>
<td>7.25</td>
<td>90.929</td>
<td>7.925</td>
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<tr>
<td>6'</td>
<td>20.77</td>
<td>96.488</td>
<td>10.086</td>
</tr>
<tr>
<td>7'</td>
<td>26.10</td>
<td>100.430</td>
<td>12.239</td>
</tr>
<tr>
<td>8'</td>
<td>29.66</td>
<td>103.795</td>
<td>14.445</td>
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<td>10'</td>
<td>34.71</td>
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<td>19.049</td>
</tr>
<tr>
<td>11'</td>
<td>35.58</td>
<td>112.397</td>
<td>21.446</td>
</tr>
<tr>
<td>12'</td>
<td>38.28</td>
<td>114.985</td>
<td>23.907</td>
</tr>
<tr>
<td>13'</td>
<td>39.83</td>
<td>117.482</td>
<td>26.428</td>
</tr>
<tr>
<td>14'</td>
<td>41.24</td>
<td>119.905</td>
<td>29.008</td>
</tr>
<tr>
<td>15'</td>
<td>42.44</td>
<td>122.266</td>
<td>31.645</td>
</tr>
<tr>
<td>16'</td>
<td>43.61</td>
<td>124.573</td>
<td>34.337</td>
</tr>
<tr>
<td>17'</td>
<td>44.68</td>
<td>126.834</td>
<td>37.083</td>
</tr>
<tr>
<td>18'</td>
<td>45.65</td>
<td>129.055</td>
<td>39.880</td>
</tr>
<tr>
<td>19'</td>
<td>46.60</td>
<td>131.239</td>
<td>42.727</td>
</tr>
</tbody>
</table>

From Fig. 8, the following relationship can be derived

\[ \theta_r = \text{angle } AOX = \text{angle } A_OX + \text{angle } A_OAa - \text{angle } A_OA \]

\[ = \frac{t}{2R_s} + \text{inv} \alpha' - \text{inv} \alpha' \] ..........................(30)

\[ t_r = 2r\theta_r = r \left[ \frac{t}{R_s} + 2 \left( \text{inv} \alpha' - \text{inv} \alpha' \right) \right] .... (31) \]

Here the tooth thickness in the pitch circle is 14.022 mm, and the pitch radius is 96 mm, and the pressure angle is 20'. As \( \theta_r \) is changeable from 1' to 20', the value \( \text{inv} \alpha' \) can be calculated using Eqs. (9) and (10), respectively. The pressure angle from pitch circle to the tip circle can be evaluated and listed in Table 1. From Eq. (11), setting \( x = r \sin \theta_r, \ y = r \cos \theta_r \), their corresponding values are listed in table 1.

3. By the PRO/E system, there is an icon to create a new curve, by "from files" selection.
It can be retrieved from the above step that had created a position file (.ibl) which is stated by the system default formation and sketched an involute curve like the one shown in Fig.9. Using the method of example 1, the gear and pinion can be constructed. The completed involute gear is shown in Fig.10.

\[ \begin{align*}
\alpha \sin \phi \cdot d\lambda &= r \sin \alpha \cdot d\phi, \\
\text{.......................... (34)}
\end{align*} \]

The involute tooth profile can be classified as a constant parametric tooth profile, the cycloidal and circular arc tooth profiles are the linear parametric tooth profiles. They are classified and listed in Table 2.

From Eqs.(32) to (34) and Table 2, they can be programmed to calculate their position values, if the gear initial condition is the same as that in example 2. It has a module of 8 mm, pressure angle of 20°, the number of teeth is 24, the diameter of the tip circle is 212 mm, and tooth thickness is 14.022 mm. The difference examples 3 and 2 is the gear contour profiles. Example 3 is changeable. It can be deducted as shown in Fig.11 to Fig.16.

**Table 2. Parametric functions for different gear types [16]**

<table>
<thead>
<tr>
<th>Item</th>
<th>Gear Type</th>
<th>Parametric function</th>
<th>Category</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Involute gear with 20° pressure angle</td>
<td>( \alpha = 1.9199 )</td>
<td>Constant</td>
<td>parametric</td>
</tr>
<tr>
<td>2</td>
<td>Cycloid gear</td>
<td>( \alpha = \frac{\pi}{2} + 3.5\phi )</td>
<td>Linear</td>
<td>parametric</td>
</tr>
<tr>
<td>3</td>
<td>Gear with a circular arc dedendum</td>
<td>( \alpha = 1.6581 + 0.5\phi )</td>
<td>constant</td>
<td>parametric</td>
</tr>
<tr>
<td>4</td>
<td>Circular arc gear with a full addendum</td>
<td>( \alpha = 1.6581 - 0.5\phi )</td>
<td>constant</td>
<td>parametric</td>
</tr>
</tbody>
</table>

Unit is radians, and the another side of gear face using \( \alpha = \pi - \alpha \)

**3-3 Example 3 : parametric tooth profiles of spur gear**

From Eq (22), the tooth profile in a planer coordinate can be presented as the following representations:

\[ \begin{align*}
\phi &= \alpha + \frac{\pi}{2} - \alpha, \\
\text{.......................... (32)}
\end{align*} \]

\[ \begin{align*}
\phi &= \alpha + \frac{\pi}{2} - \alpha, \\
\text{.......................... (33)}
\end{align*} \]

The constrained equation is
4. Conclusion

Gearing theory is an interesting and attractive study topic that strongly draws the attention of authors. In this paper, spur gear profiles and their theories are introduced and deduced, and three examples are illustrated to authenticate. It is clear that the parametric tooth model, the polynomial parametric function, the coefficients and the degrees of the parametric function affect the geometric contours and meshing properties of gears. The investigated kinematics characteristics of the parametric functions are illustrated in example 3. We intend to attract the reader to use the computer solid modeling system to aid the gear design and show the unambiguous solid prototypes. These solid prototypes are very important for manufacturing, motion simulation and further studies on contact.
mechanics are required. Gear is not only the spur gear, it has various types of gears. Spur gear is the most useful and simple type of gears. The different types of gears are helical gear (for two parallel rotating axes), bevel and conical involute gear (for two nonparallel or coplanar axes), crossed helical gear, cylindrical worm gear, hypoid, spiroid, and helicon gear (for two nonparallel, noncoplanar axes). It is a very interesting topic and the authors will keep the study going.

Reference


